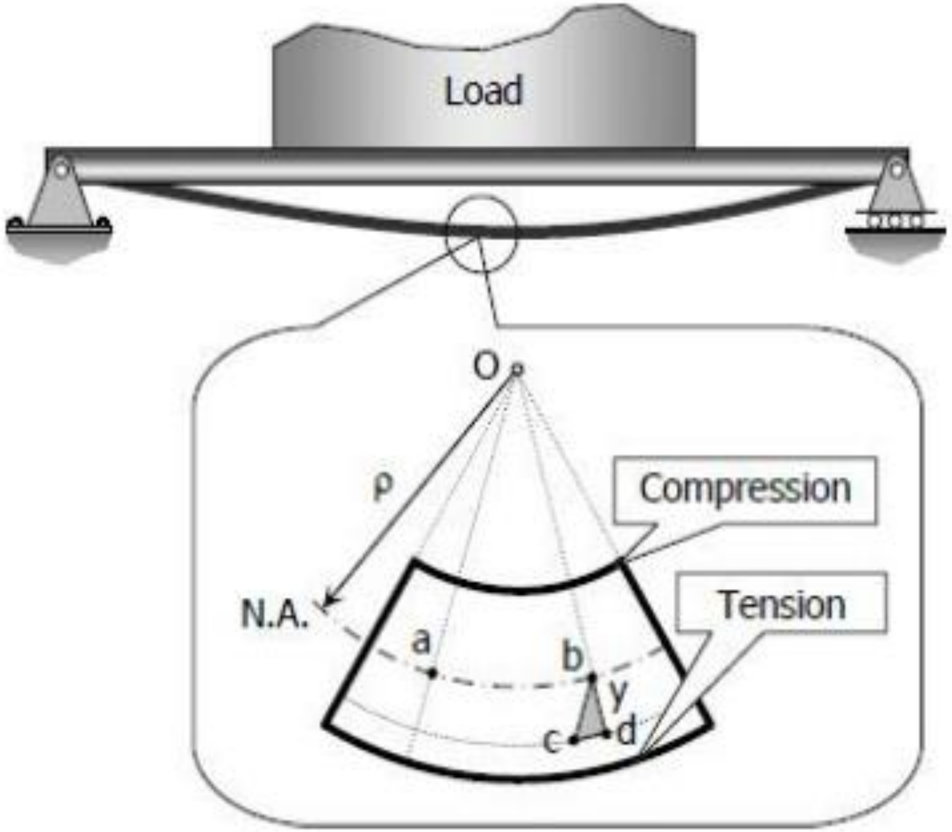
# Stresses in Beam

**Flexure Formula**

Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown.



Consider a fiber at a distance y from the neutral axis, because of the beam's curvature, as the effect of bending moment, the fiber is stretched by an amount of cd. Since the curvature of the beam is very small, bcd and Oba are considered as similar triangles. The strain on this fiber is

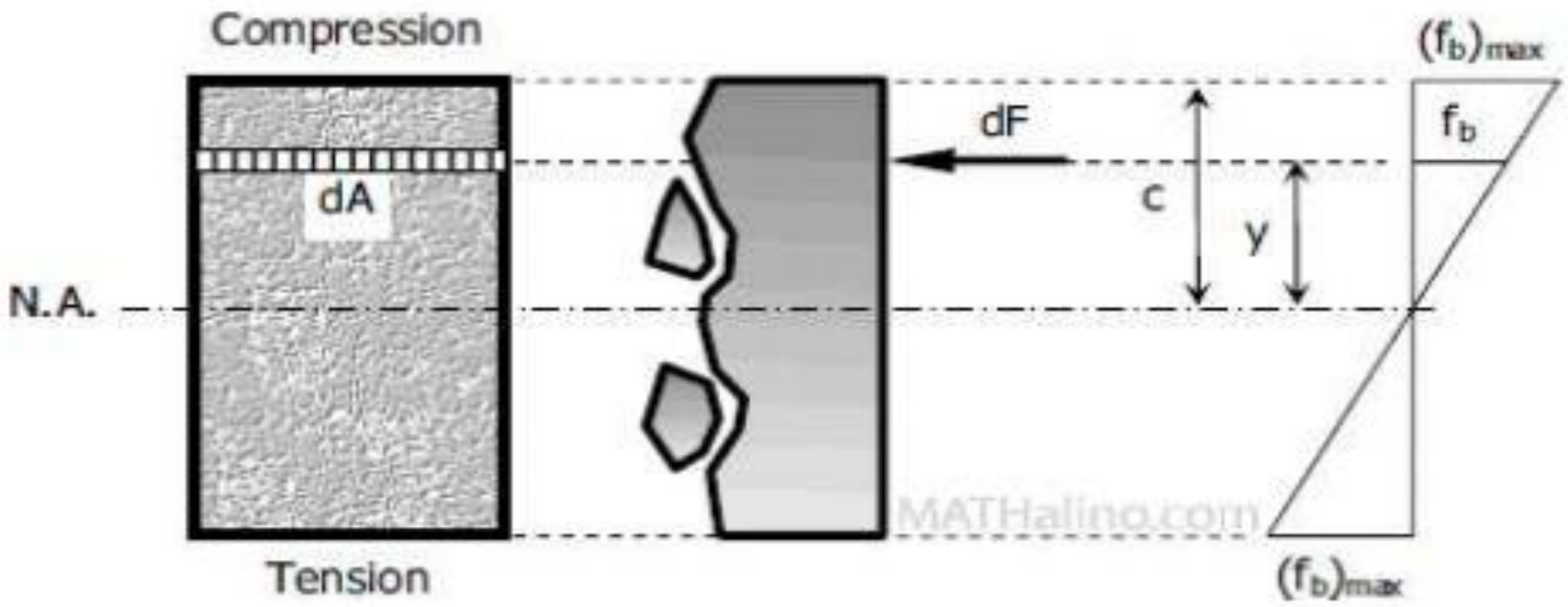
$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho}$$

By Hooke's law, $\varepsilon = \sigma / E$, then

$$\frac{\sigma}{E} = \frac{y}{\rho}; \ \sigma = \frac{y}{\rho} E$$

which means that the stress is proportional to the distance y from the neutral axis.

**For this section, the notation $f_b$ will be used instead of $\sigma$.**




**Considering a differential area dA at a distance y from N.A., the force acting over the area is**

$$dF = f_b\, dA = \frac{y}{\rho} E\, dA = \frac{E}{\rho} y\, dA$$

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

$$M = \int dM = \int y\,dF = \int y\left(\frac{E}{\rho} y\,dA\right)$$

$$M = \frac{E}{\rho}\int y^2\,dA$$

but $\int y^2\,dA = I$ , then

$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M} \quad \Longrightarrow \quad \text{substituting } \rho = Ey/f_b \quad \Longrightarrow \quad \frac{Ey}{f_b} = \frac{EI}{M}$$

Then $$f_b = \frac{My}{I}$$ and $$(f_b)_{max} = \frac{Mc}{I}$$

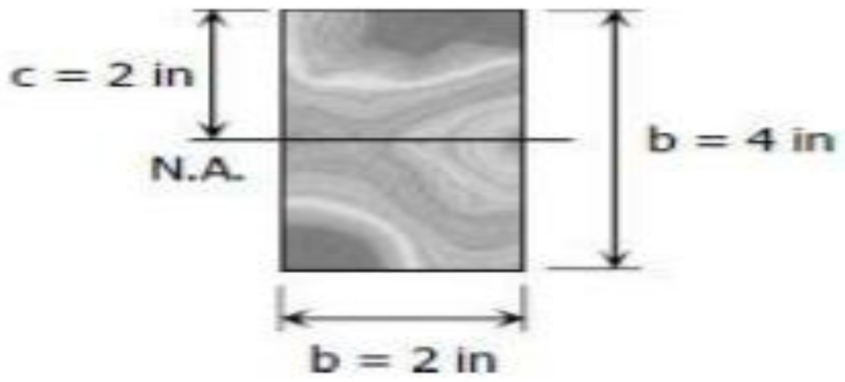
# Example 1

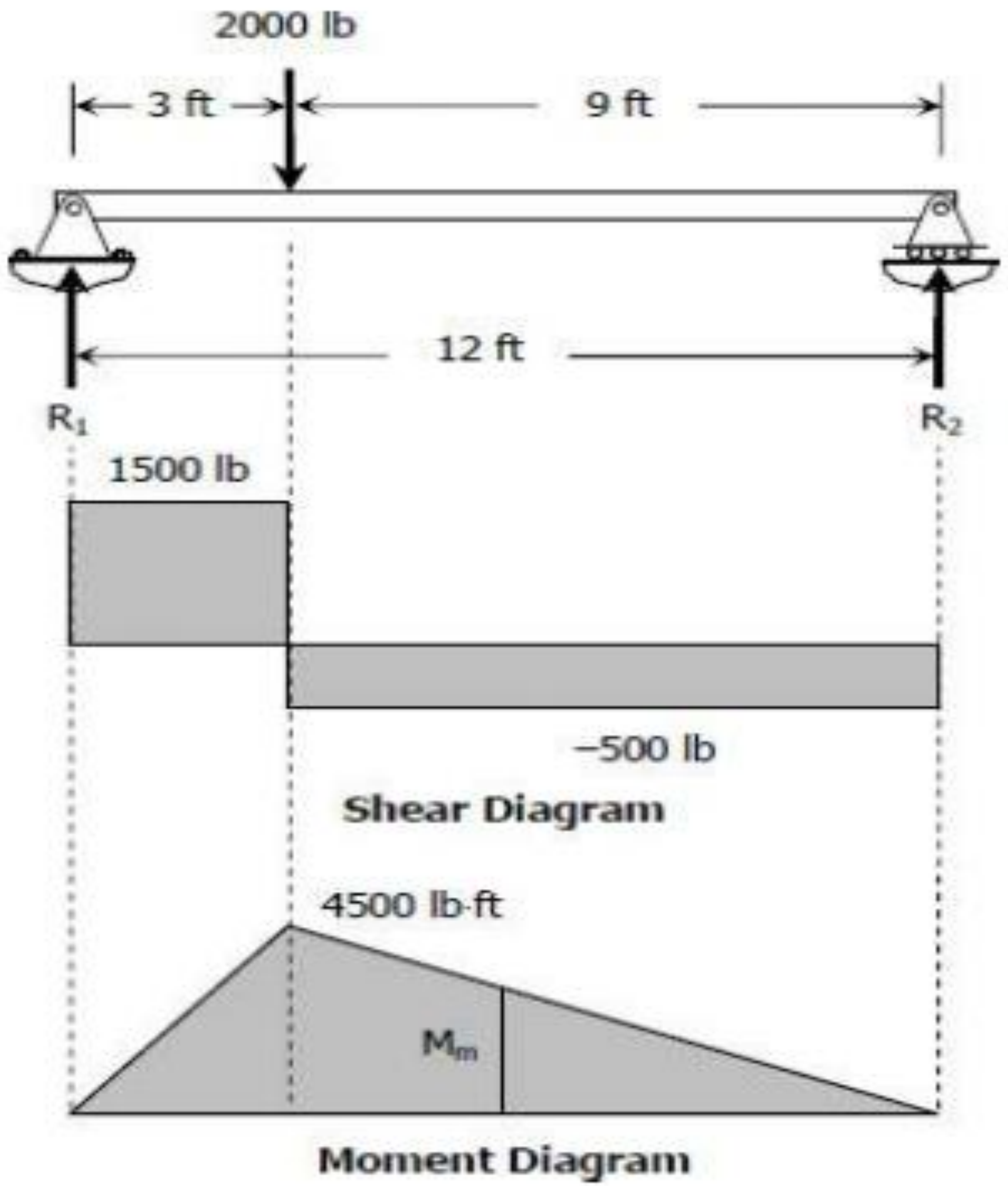
A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to a concentrated load of 2000 lb at a point 3 ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at midspan.

$$\Sigma M_{R2} = 0$$
$$12R_1 = 9(2000)$$
$$R_1 = 1500\,\text{lb}$$

$$\Sigma M_{R1} = 0$$
$$12R_2 = 3(2000)$$
$$R_2 = 500\,\text{lb}$$

**Maximum fiber stress:**



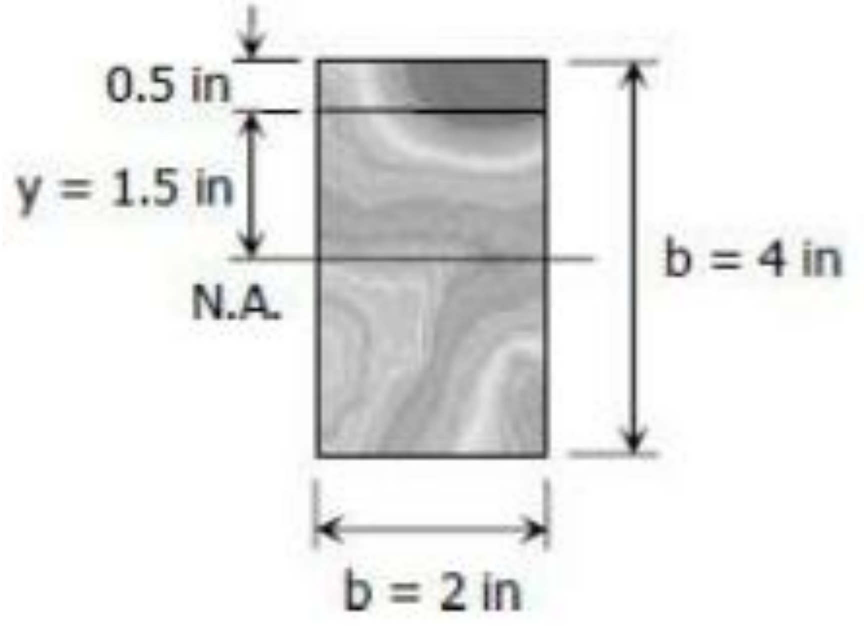
$$(f_b)_{max} = \frac{Mc}{I} = \frac{4500(12)(2)}{\frac{2(4^3)}{12}}$$

$$(f_b)_{max} = 10,125 \text{ psi} \qquad answer$$

**Stress in a fiber located 0.5 in from the top of the beam at midspan:**

$$\frac{M_m}{6} = \frac{4500}{9}$$

$$M_m = 3000\,\text{lb} \cdot \text{ft}$$




$$f_b = \frac{My}{I}$$

$$f_b = \frac{3000(12)(1.5)}{\frac{2(4^3)}{12}}$$

$$f_b = 5,062.5 \text{ psi} \qquad answer$$